given above. For convenience, since this value was used earlier for water, $n = 3$ was adopted and the value of 4·28 for $K_3$ was obtained by minimization of the sum of the squares of residuals of ln ($\alpha$).

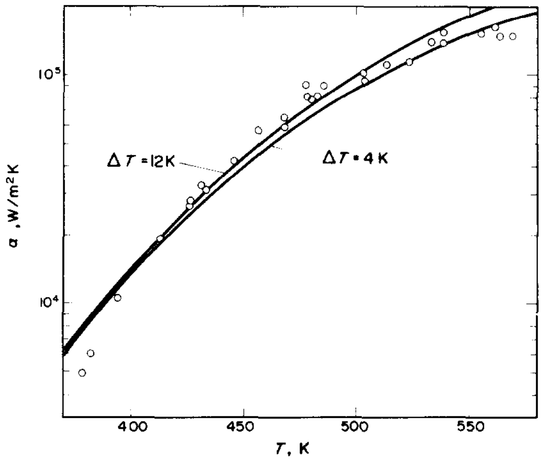


FIG. 1. Comparison between theory and measurements [11].

## 4. CONCLUDING REMARKS

It may be seen from Fig. 1 that the theory indicates a small increase in $\alpha$ with $\Delta T$ where the experiments showed such an increase only for $\Delta T$ less than about 2 K. Also, the curvature of the theoretical lines is not so great as that exhibited by the experimental points. However, it is clear from Fig. 1 that the theory and observations are in fair agreement over the whole range of $T$ and $\Delta T$ when the constants do not differ widely from their expected values. In view of the wide differences in properties, notably thermal conductivity, between water and mercury, the theory receives strong support from the fact that it is found to be generally satisfactory in both cases.

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*Int. J. Heat Mass Transfer.* Vol. 15, pp. 1434–1439. Pergamon Press 1972 Printed in Great Britain

# THE RESPONSE OF HEATED WIRE TEMPERATURE DISTRIBUTIONS TO UNSTEADY SURFACE COOLING RATES

M. R. DAVIS

School of Mechanical and Industrial Engineering, The University of New South Wales, Kensington, N.S.W. 2033, Australia

(*Received* 12 *July* 1971 *and in revised form* 7 *February* 1972)

## NOMENCLATURE

$A$, feedback parameter ($dR_w/di$);
$c$, specific heat of wire material;
$d$, wire diameter;
$E$, bridge voltage;
$h$, surface heat transfer coefficient;
$i$, wire heating current;
$k$, thermal conductivity of wire material;
$2l$, wire total length;
$R_w$, wire total resistance;
$R_1, R_2, R_3$, resistance of feedback network resistors;
$t$, time;
$T$, wire local temperature;
$T_f$, flow temperature;
$T_0$, ambient temperature;
$x$, position on wire ($x = 0$ at centre);
$\rho$, density of wire material;
$\rho_T$, resistivity of wire material;
$\tau_1$, time constant of heating current response.

## 1. INTRODUCTION

THE STEADY distribution of temperature along heated wires of large aspect ratio has been analyzed by King [1] and other investigators [2–5]. Davies and Fisher [6] used numerical integration to match the steady state boundary conditions satisfactorily. The dynamic response of constant resistance systems has been discussed by Davis [7], introducing the effect of probe inductance, whilst other authors [8–10] have considered the unsteady response of the wire or composite cross section temperature distribution. The present work deals with the unsteady lengthwise distribution of temperature in high aspect ratio wires heated by a constant resistance amplifier, neglecting the effect of probe inductance.

## 2. SOLUTIONS FOR THE STEADY AND UNSTEADY TEMPERATURE DISTRIBUTION

(i) *Steady state solutions for the local wire temperatures*

The thermal equilibrium of a relatively long circular cylinder heated by a current $i$, with a coefficient of heat transfer $h$ to the surrounding fluid at temperature $T_f$, is determined by the differential equation for $T(x)$, if variations across the cross section are neglected. The equation is

$$\frac{\pi d^2}{4}\cdot\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{4i^2\rho_T}{\pi d^2} - \pi d h(T - T_f) - \frac{\rho c \pi d^2}{4}\,\frac{\partial T}{\partial t}\Bigg) = 0 \qquad (1)$$

where $k$, $\rho_T$ and $h$ are the wire thermal conductivity, resistivity and surface convective heat transfer coefficient respectively, all of which may vary significantly with the wire temperature. The wire is of density $\rho$, diameter $d$, which is assumed to be constant along its length ($-l < x < l$). Thermoelectric and radiative heat transfer effects were found to be quite negligible and are omitted from this energy balance equation. Under conditions of steady equilibrium, solutions to equation 1 may be made by numerical integration, iterating for the unknown heat transfer coefficient and end temperature gradient ($\partial T/\partial x$ at $x = -l$) in order to satisfy the overall energy balance,

$$i^2 R_w = \frac{k\pi d^2}{4}\left(\left(\frac{\partial T}{\partial x}\right)_{+l} - \left(\frac{\partial T}{\partial x}\right)_{+l}\right) + \pi d\int_{-l}^{l} h(T - T_f)\,dx$$

and integrated electrical resistance along the wire $\left(R_w = \int_{-l}^{l}\frac{4\rho_T}{\pi d^2}\,dx\right)$ and to match the boundary conditions of temperature at the wire ends ($T_0$ at $x = \pm l$). The integration may be carried out by a simple step-by-step procedure using a forward difference method, commencing at $x = -l$, and the required conditions can be matched to within 0·1 per cent. The fluid temperature $T_f$ is assumed constant and equal to ambient temperature $T_0$ all along the wire. The following properties of tungsten were used:

$$\rho = 18{\cdot}5\ \text{g/cm}^3, \quad c = 0{\cdot}032\ \text{cal/g}^\circ\text{C},$$
$$\rho_T = (5{\cdot}2\,\tau + 0.21\tau^2)\,10^{-6}\ \Omega\,\text{cm},$$
$$k = 6{\cdot}92\,(T/\rho_T)\,10^{-6}\ \text{W/cm}^2\,{}^\circ\text{K}$$

where $\tau$ is a non-dimensional temperature introduced by Davis and Fisher [6] on the basis of the wire resistance as $\tau = (T - 54)/237$. It is found that the resulting steady distributions of temperature are flatter at higher cooling rates, the temperature at the wire centre increasing with decreasing surface cooling rate whilst the average temperature and electrical resistance remain constant.

(ii) *Unsteady state solutions for the wire temperature*

A typical constant resistance feedback system is shown in Fig. 1. For this system, elementary network analysis shows that

$$R_w' = E'\left\{\frac{R_1 + \bar{R}_w\left(\dfrac{1}{G} - \dfrac{\bar{e}}{\bar{E}}\right)}{\bar{E}\left(\dfrac{R_1}{R_1 + \bar{R}_w}\right) + E'\left(\dfrac{R_2}{R_2 + R_3} - \dfrac{1}{G}\right)}\right\} \qquad (2)$$

where an overbar denotes a mean value and primed variables represent fluctuations. For small values of $(E'/\bar{E})$ this may be approximated by

$$R_w' = A_{i'} \qquad (3)$$

where

$$A = \frac{\left(\dfrac{1}{G} - \dfrac{\bar{e}}{\bar{E}}\right)(R_1 + \bar{R}_w)^3}{\bar{E}R_1}\,. \qquad (4)$$

For a 2 mm long, 5 μm dia. tungsten wire, the variation of mean temperature is typically less than 5°C. It is found that the redistribution temperature variations are much larger, having a maximum value of $\pm 40$°C at $x/l = 0, \pm 0{\cdot}75$.

Solutions to equation (1) for unsteady cooling have been carried out for the case where an initial equilibrium is disturbed by a step change of surface heat transfer coefficient.

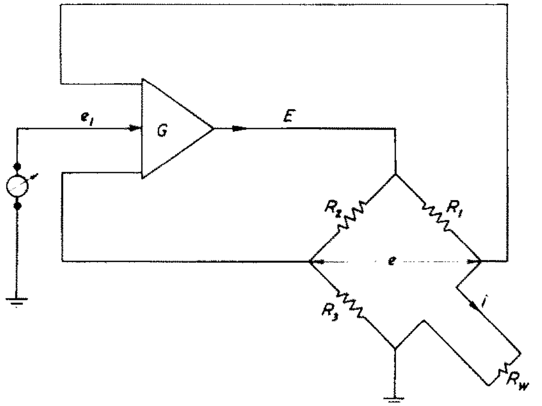


FIG. 1. Constant resistance feedback heating system.

The solutions for $T(x, t)$ were carried out by numerical integration beginning with the known initial steady state solution $T(x, 0)$. Values of $\partial T/\partial t$ were evaluated along the wire from equation (1), the heat transfer coefficient being changed to its new value but all other variables remaining unaltered. The heating current was increased from its initial value by evaluating the instantaneous rate of change of total wire resistance,

$$\frac{dR_w}{dt} = \frac{4}{\pi d^2} \int_{-l}^{l} \frac{\partial \rho_T}{\partial T} \cdot \frac{\partial T}{\partial t} \cdot dx \tag{5}$$

and for each small increment of time $\delta t$ the current was increased using the equation

$$i(t + \delta t) = i(t) + \frac{1}{A} \frac{dR_w}{dt} \delta t. \tag{6}$$

Similarly, the local wire temperature was modified using the equation for the small interval of time

$$T(x, t + \delta t) = T(x, t) + \frac{\partial T}{\partial t} \delta t. \tag{7}$$

It was necessary to use a logarithmically increasing time step size ($\delta t$) in order to complete the solutions without excessively long computational times. It was found that the heating current during the transient approached the final steady state condition smoothly.

The transient response of the wire heating current is shown in Fig. 2 which shows that the response is of approximately exponential form with a time constant determined by the value of $A$. In Fig. 3 the variation of the current response time constant is plotted as a function of the

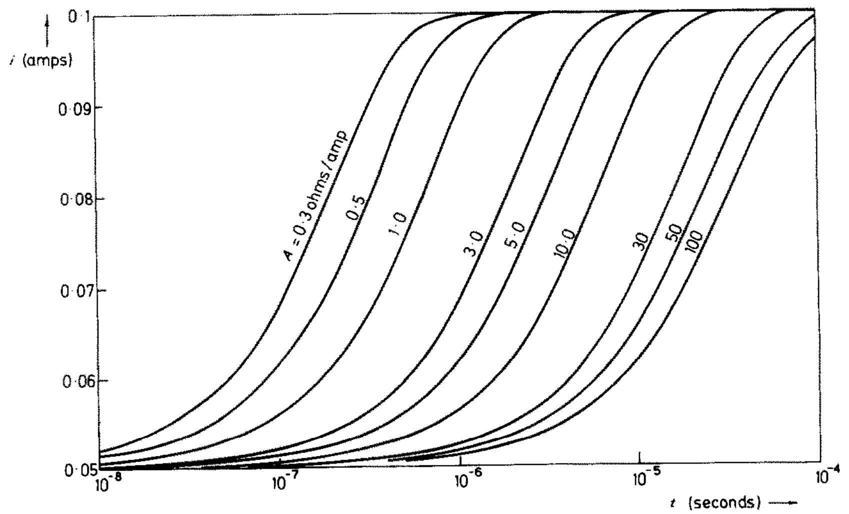


FIG. 2. Response of heating current to a step change of surface heat transfer coefficient.

Wire: Tungsten, length 0·194 cm, dia. = 0·00051 cm, resistance 14·97 Ω.

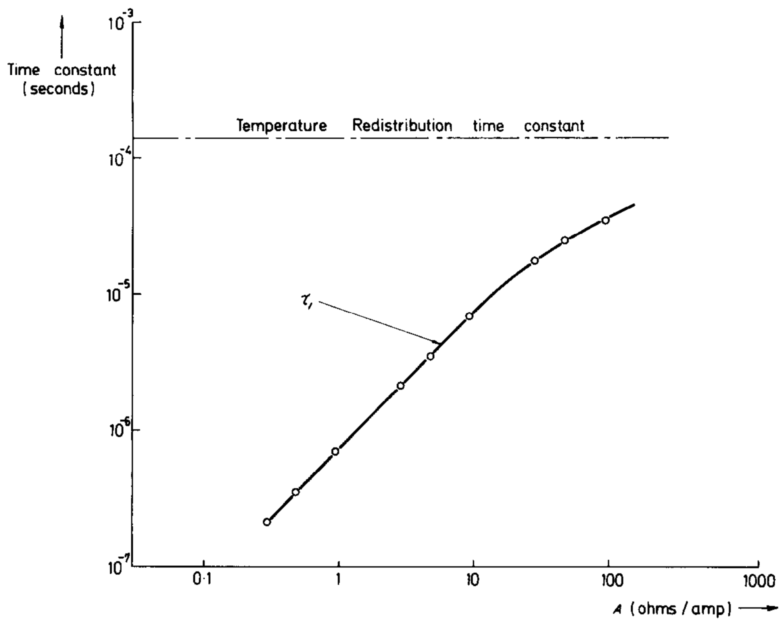


FIG. 3. Variation of time constant with feedback gain parameter.

constant $A$ and it may be seen that the two are linearly related except for cases where $A$ is relatively large. It was found that the response to positive and negative step changes in heat transfer was similar.

The dynamic response of the heating feedback loop may be considered more simply by representing the wire in terms of its total thermal capacity, neglecting the variation of temperature along the wire. For relatively small variations of current ($i'$) about the mean value ($\bar{i}$) the unsteady overall energy balance equation,

$$\rho c \,.\, 2l \,.\, \frac{\pi d^2}{4} \,.\, \left(\frac{\mathrm{d}T_{av}}{\mathrm{d}R_w}\right)\left(\frac{\mathrm{d}R_w}{\mathrm{d}t}\right) + \pi d \,.\, 2l \,.\, h(T_{av} - T_f) + K = i^2 R_w \qquad (8)$$

may be written, after subtracting the mean energy balance equation and neglecting second order products of fluctuating terms, as

$$\left(-\frac{\rho c \pi^2 d^4}{16\bar{E}(\partial \rho_T/\partial T)}\right)\left(\frac{\mathrm{d}R_w}{\mathrm{d}i}\right)\frac{\mathrm{d}i'}{\mathrm{d}t} + 2\bar{i}\,\bar{R}_w\, i = \pi d \,.\, 2l\, h'_{av}(T_{av} - T_0). \qquad (9)$$

It is assumed here that $R_1 = \bar{R}_w$, so that $\bar{E} = \bar{i}(R_1 + \bar{R}_w)$. Variations in the conduction loss to the wire ends ($K$) have been neglected and average quantities along the wire are denoted by the suffix "av". This equation represents the first order response of the heating current to unsteady variations of the average heat transfer coefficient ($h'_{av}$) and has a time constant $\tau_1$ given by

$$\tau_1 = \left(-\frac{\rho c \pi^2 d^4}{16\bar{E}(\partial \rho_T/\partial T)}\right) \cdot \left(\frac{\mathrm{d}R_w}{\mathrm{d}\bar{i}}\right). \qquad (10)$$

Substitution of the properties of a $5\,\mu$ dia. tungsten wire into this equation with a value of $\mathrm{d}R_w/\mathrm{d}i = -3\,\Omega/\mathrm{A}$ gives a value for $\tau_1 = 2{\cdot}19 \times 10^{-6}$ s. This compares quite closely with the value of $2{\cdot}10 \times 10^{-6}$ s obtained for the numerical solution shown in Fig. 2. The small difference between these two results would be caused by the assumptions made in writing the averaged equation for the unsteady energy balance, neglecting changes in $K$ (equation (8)). From these results, it appears that the simplified model of the wire in terms of its overall averaged energy balance, as represented in equation (9). gives a close prediction of the transient response of the feedback heating loop.

The response of the local wire temperature during the transient following the step change of applied cooling is shown in Fig. 4 for two positions on the wire. It may be seen that the transient variation of temperature is composed of two parts, one due to the change in mean temperature required to activate the feedback loop and the other due to the process of redistribution of the wire temperature into the steady state condition corresponding to the new rate of heat transfer from the wire surface. The latter part of the transient takes place with a fixed time constant

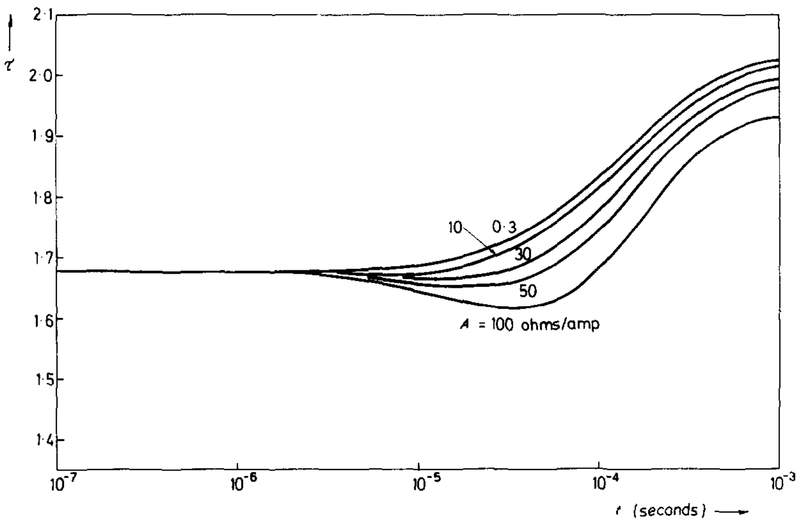


FIG. 4a. Response of local wire temperature to change of surface heat transfer ($x/l = \pm 0{\cdot}75$).

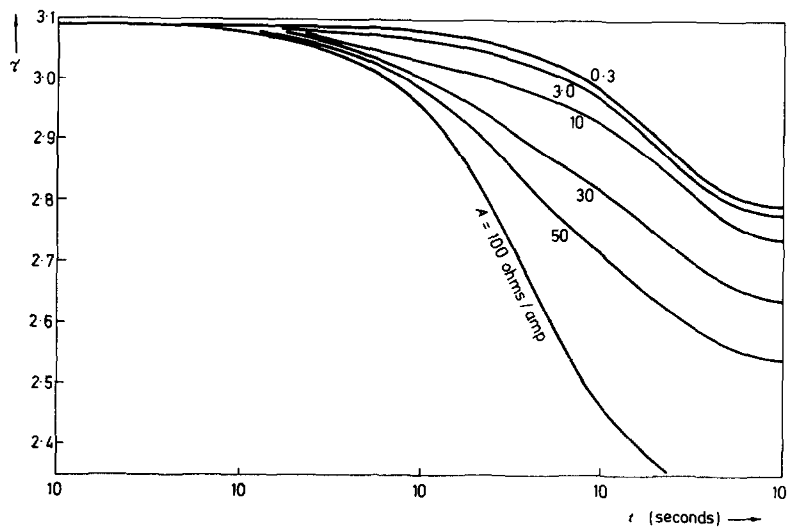


FIG. 4b. Response of local wire temperature to change of surface heat transfer ($x/l = 0$).

of $140 \times 10^{-6}$ s for all the responses shown and leads to a nett decrease or increase of local temperature after the transient, consistent with the change to a flatter distribution of temperature at the higher cooling rate. The first part of the transient is a reduction of temperature due to the reduction of overall resistance and average temperature associated with the increase of heating current by the feedback loop. The time constant of this initial part of the temperature transient is identical with that of the heating current response, the magnitude of this temperature change decreasing for the faster responses where the wire resistance change is smaller.

## 3. CONCLUSIONS

Neglecting inductive and other higher order effects in the feedback heating loop, solutions to the unsteady response of a heated wire maintained at almost constant average temperature and electrical resistance have shown that the time constant of the heating current response is determined by the overall wire thermal capacity and by the conditions of the feedback network and heating amplifier. The local temperature changes which contribute to the average wire temperature take place with this same time constant and are of the same sense (positive or negative depending on the change of cooling) all along the wire length.

The variation of temperature distribution between different surface cooling rates gives rise to flatter distributions of temperature at higher cooling rates and this effect superimposes a second component of variation to local wire temperatures during unsteady cooling. These changes take place independently of the response of the heating current, although they can be considerably larger than the change of average wire temperature which activates the feedback network and amplifier. It may be concluded that the response of the wire heating current is not limited by the redistribution of temperature associated with a change of average cooling rate.

For a tungsten wire 2 mm long and 5 μ dia., it was found that the redistribution temperature changes took place with a time constant of 140 μ s. For this particular wire it would therefore be expected that the distribution of temperature would not respond to variations of cooling at frequencies in excess of 1·1 kHz. The wire would then retain the temperature distribution corresponding to the average heat loss rate, only experiencing the small uniformly distributed variations of temperature along the wire necessary to alter the wire resistance and control the feedback heating amplifier. For fluctuations of cooling at lower frequencies, the temperature distribution would respond to the changes in surface heat transfer and the wire would then maintain a quasi-steady equilibrium during the unsteady cooling variations. Since the redistribution of temperature does not influence the response of the heating current, there will be no direct modification introduced to the calibrations of hot wires used in turbulent flow measurements as a consequence of this effect.

**ACKNOWLEDGEMENT**

The support of the Science Research Council during the progress of this work is gratefully acknowledged.

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